Section 2.2 Derivatives of Products and Quotients (Minimum Homework: all odds)

#1-12: Use the product rule to find the derivatives of the following.

1) $y = (2x + 3)(3x - 4)$

First factor $2x + 3$	Second Factor $3x - 4$
Derivative 2	Derivative 3
cross multiply top down $(2x + 3)(3) = 6x + 9$	*cross multiply bottom up* $2(3x - 4) = 6x - 8$

$$y' = 6x + 9 + 6x - 8$$

$$y' = 6x + 6x + 9 - 8$$

$$y' = 12x + 1$$

Answer: $y' = 12x + 1$

3) $f(x) = (x - 2)(3x - 4)$

First factor $x-2$	Second Factor $3x-4$
Derivative 1	Derivative 3
cross multiply top down $(x-2)3 = 3x-6$	*cross multiply bottom up* $1(3x-4) = 3x-4$

$$f'(x) = 3x-6+3x-4$$

$$f'(x) = 3x+3x-6-4$$

$$f'(x) = 6x-10$$

Answer: $f'(x) = 6x - 10$

5) $f(x) = (x^2 + 3x + 2)(3x - 5)$

First factor $x^2 + 3x + 2$	Second Factor $3x - 5$
Derivative $2x + 3$	Derivative 3
cross multiply top down $(x^2 + 3x + 2)3 = 3x^2 + 9x + 6$	*cross multiply bottom up* $(2x + 3)(3x - 5) = 6x^2 - 1x - 15$

$$f'(x) = 3x^2 + 9x + 6$$

$$+ \quad 6x^2 - 1x - 15$$

$$f'(x) = 9x^2 + 8x - 9$$

Answer: $f'(x) = 9x^2 + 8x - 9$

7) $g(t) = (2t - 1)(3t + 5)$

First factor $2t - 1$	Second Factor $3t + 5$
Derivative 2	Derivative 3
$cross\ multiply\ top\ down\ \ 2(3t+5) = 6t + 10$	$cross\ multiply\ bottom\ up$ $(2t-1)(3) = 6t - 3$

$g'(T) = 6T + 10 + 6T - 3$

$g'(T) = 6T + 6T + 10 - 3$

$g'(T) = 12T + 7$

$answer\ g'(t) = 12t + 7$

9) $y = 3x^2(2x^2 + 6x - 4)$

First factor $3x^2$	Second Factor $2x^2 + 6x - 4$
Derivative $6x$	Derivative $4x + 6$
cross multiply top down $3x^2(4x + 6) = 12x^3 + 18x^2$	*cross multiply bottom up* $6x(2x^2 + 6x - 4) = 12x^3 + 36x^2 - 24x$

$$\frac{dy}{dx} = 12x^3 + 18x^2$$
$$+ 12x^3 + 36x^2 - 24x$$

$$\frac{dy}{dx} = 24x^3 + 54x^2 - 24x$$

gcF $6x$

Answer: $\frac{dy}{dx} = 6x(4x^2 + 9x - 4)$

$$\frac{dy}{dx} = 6x(4x^2 + 9x - 4)$$

11) $y = (3x^4)(5x^2 + 7)$

First factor $3x^4$	Second Factor $5x^2 + 7$
Derivative $12x^3$	Derivative $10x$
cross multiply top down $3x^4(10x) = 30x^5$	*cross multiply bottom up* $12x^3(5x^2 + 7) = 60x^5 + 84x^3$

$\frac{dy}{dx}$ or $y' = 30x^5 + 60x^5 + 84x^3$

$y' = 90x^5 + 84x^3$

GCF $6x^3$

$y' = 6x^3(15x^2 + 14)$

Answer: $y' = 6x^3(15x^2 + 14)$

#13-20: Use the quotient rule to find the derivative of the following.

13) $f(x) = \frac{6}{5x+1}$

Denominator $5x + 1$	Numerator 6
Derivative 5	Derivative 0
$cross\ multiply\ top\ down$ $(5x + 1)(0) = 0$	$cross\ multiply\ bottom\ up$ $5 * 6 = 30$

$$f'(x) = \frac{0 - 30}{(5x+1)^2}$$

$$f'(x) = \frac{-30}{(5x+1)^2}$$

Answer: $f'(x) = \frac{-30}{(5x+1)^2}$

15) $y = \frac{9x}{x-5}$

Denominator $x - 5$	Numerator $9x$
Derivative 1	Derivative 9
cross multiply top down $(x-5)(9) = 9x - 45$	*cross multiply bottom up* $1(9x) = 9x$

$$\frac{dy}{dx} = y' = \frac{9x - 45 - 9x}{(x-5)^2}$$

$$y' = \frac{-45}{(x-5)^2}$$

Answer: $y' = \frac{-45}{(x-5)^2}$

17) $y = \frac{3t+1}{2t+5}$

Denominator $2t + 5$	Numerator $3t + 1$
Derivative 2	Derivative 3
cross multiply top down $(2t + 5)(3) = 6t + 15$	*cross multiply bottom up* $2(3t + 1) = 6t + 2$

$$y' \text{ or } \frac{dy}{dT} = \frac{6T+15-(6T+2)}{(2T+5)^2}$$

$$\frac{dy}{dt} = \frac{6T+15-6T-2}{(2T+5)^2}$$

$$\frac{dy}{dt} = \frac{13}{(2T+5)^2}$$

Answer: $\frac{dy}{dt} = \frac{13}{(2t+5)^2}$

19) $g(x) = \frac{x^2}{x-4}$

Denominator $x - 4$	Numerator x^2
Derivative 1	Derivative $2x$
$cross\ multiply\ top\ down$ $(x-4)(2x) = 2x^2 - 8x$	$cross\ multiply\ bottom\ up$ x^2

$$g'(x) = \frac{2x^2 - 8x - 1x^2}{(x-4)^2}$$

$$g'(x) = \frac{1x^2 - 8x}{(x-4)^2}$$

$$g'(x) = \frac{x(x-8)}{(x-4)^2}$$

$answer$: $g'(x) = \frac{x(x-8)}{(x-4)^2}$

#21-26:

a) Find the slope of the tangent line to the graph of the function for the given value of x (or t).

b) Find the equation of the tangent line to the graph of the function for the given value of x (or t).

21) $y = (2x + 3)(3x - 4);\ x = 2$

(derivative computed in #1)

a) $y' = 12x + 1$

$m = 12(2) + 1 = 25$

Answer: $m = 25$

b) y-coordinate $y = (2 * 2 + 3)(3 * 2 - 4) = 14$

$$point\ \ (2, 14) \quad slope\ \ m = 25$$

$$y - 14 = 25(x - 2)$$

$$y - 14 = 25x - 50$$

$$+14 \qquad\qquad +14$$

$$y = 25x - 36$$

Answer: $y = 25x - 36$

#21-26:

a) Find the slope of the tangent line to the graph of the function for the given value of x (or t).

b) Find the equation of the tangent line to the graph of the function for the given value of x (or t).

23) $g(t) = (2t - 1)(3t + 5)$;t = 4

(derivative computed in #7)

a) $g'(t) = 12t + 7$

$m = g'(4) = 12 * 4 + 7 = 55$

$answer\ \ m = 55$

b) y-coordinate: $y = g(4) = (2 * 4 - 1)(3 * 4 + 5) = 119$

$point\ \ (4, 119)\quad slope\ \ m = 55$

$y - 119 = 55(x - 4)$

$y - 119 = 55x - 220$

$+119 \qquad\qquad +119$

$y = 55x - 101$

$answer\ y = 55x - 101$

#21-26:

a) Find the slope of the tangent line to the graph of the function for the given value of x (or t).

b) Find the equation of the tangent line to the graph of the function for the given value of x (or t).

25) $f(x) = \frac{6}{5x+1}$; $x = 1$

(derivative computed in #13)

a) $f'(x) = \frac{-30}{(5x+1)^2}$

$m = f'(1) = \frac{-30}{(5(1)+1)^2} = -\frac{30}{36} = -\frac{5}{6}$

$answer\ \ m = -\frac{5}{6}$

b) y-coordinate $y = f(1) = \frac{6}{5*1+1} = 1$

$point\ (1,1)\ slope\ \ m = -5/6$

$$y - 1 = \frac{-5}{6}(x-1)$$

$$y - 1 = \frac{-5}{6}x + \frac{5}{6}$$

$$+1 \qquad\qquad +1$$

$$y = \frac{-5}{6}x + \frac{11}{6}\ *$$

Answer: $y = -\frac{5}{6}x + \frac{11}{6}$

$$* \quad \frac{5}{6} + 1 = \frac{5}{6} + \frac{1(6)}{1(6)}$$

$$= \frac{5}{6} + \frac{6}{6} = \frac{11}{6}$$